Theoretical Notes
Note 40

MEMORANDUM
RM-4306
OCTOBER 1964

DETECTION OF THE ELECTROMAGNETIC RADIATION FROM NUCLEAR EXPLOSIONS IN SPACE

W. J. Karzas and Richard Latter

The RAND Corporation
1700 MAIN ST. • SANTA MONICA • CALIFORNIA • 90406

PREFACE

The present report is part of a continuing effort to find improved methods for ground–based detection of nuclear explosions conducted at high altitudes and in space.

SUMMARY

The electromagnetic signal generated by interaction of nuclear–explosion–induced electric currents with the geomagnetic field can serve as a basis for detecting such explosions at extreme distances from the earth. In the present paper, an earlier theory of this phenomenon developed by the authors is extended to allow estimates of the maximum detection range. It is concluded that this electromagnetic signal may be detectable at distances of the order of $10^7 \sqrt{Y_{KT}}$ kilometers from a nuclear explosion of total yield Y_{KT} kilotons.

CONTENTS

I. INTRODUCTION

In a previous paper[1], the authors considered the detection of electromagnetic radiation from a nuclear explosion in space caused by Compton–recoil electrons from gamma rays scattered in the materials of the nuclear device or in a surrounding material shield— the so–called case signal. It was concluded that the case signal had a peak electric field strength of about $10^4\ R^{-3/2}$ v/m at a distance R kilometers from the explosion. The maximum detectable range of this signal was estimated to be 10^6 km, independent of the explosion yield.

More recently[2], the authors considered another mechanism for the generation of electromagnetic radiation from nuclear explosions which might be useful for detection. This mechanism was based on the interaction of nuclear explosion–induced electric currents with the geomagnetic field. The theory of this mechanism was applied to explosions which take place well within the earth's atmosphere. It was indicated, however, that this same geomagnetic interaction mechanism applies to high–altitude explosions where it can lead to

---

[1] W. J. Karzas and R. Latter, Phys. Rev. 126, 1919 (1962).

[2] W. J. Karzas and R. Latter, Journ. of Geoph. Res. 67, 4635 (1962).

quite intense electromagnetic signals at great distances from the burst point—thereby suggesting its usefulness for detecting nuclear explosions in space. In the present paper, this earlier theory of the geomagnetic interaction mechanism will be extended so as to permit an estimate of the intensity of these electromagnetic signals observed on the earth's surface from nuclear explosions conducted at the great distances of interest to the detection problem. It is shown that this signal can be detected at significantly greater distances than the case signal.

II. GENERAL CONSIDERATIONS

For nuclear explosions below an altitude of 100 or so kilometers the primary source of the electromagnetic signal is Compton-recoil electrons produced by explosion gamma rays interacting with the atmosphere. Above 100 or so kilometers, x rays[3] from the heated materials of the nuclear device generate photoelectrons which also contribute a radiating current.

The currents produced by the gamma rays and x rays from high-altitude explosions have maxima in two regions. One region is near the explosion where the gamma-ray and x-ray intensities are highest even though in this region the air density may be quite low. The second region is in the earth's atmosphere at altitudes of about 20 to 40 or so kilometers for gamma rays and of about 70 to 110 or so kilometers for x rays. These altitudes correspond to the region of peak absorption for explosion gamma rays which have a mean energy of about one mev and for explosion x rays which have a Planck distribution corresponding to a nominal temperature of about one kev.

In our treatment of the detection problem, radiation from the currents near the burst point will be neglected. For explosions at

---

[3]At lower altitudes, the range of the x-ray photoelectrons is too short to contribute an appreciable current. Moreover, the x rays are confined to a relatively small region about the burst point, which region is made conducting by ionization from the earlier emission of gamma rays and so cannot radiate.

altitudes which are not too great, the ionization in the atmosphere produced by the gamma rays and x rays is sufficient to absorb this radiation before it reaches the earth's surface. For explosions at extreme altitudes, where the gamma-ray and x-ray ionization is negligible, this radiation can reach the earth's surface, but it may still be neglected, since its intensity turns out to be substantially less than that from the atmospheric currents. This lower intensity results from the very long mean free path for production of currents near the burst point, which more than compensates for the closeness of this region to the burst compared to the great distance of the atmosphere. Moreover, the intensity of the signal from the region near the burst point is further degraded by dispersion through having to traverse the low density ionized space medium for great distances and then having to traverse the high density ionized ionosphere.

Figure 1 shows schematically the geometry of the explosion. Since the currents contributing to the electromagnetic field at the observation point are confined to a relatively small region near that point, this geometry may be simplified by assuming that the geomagnetic field is uniform (See Fig. 2). Spherical polar coordinates will be used. The origin will be located at the burst point and the polar axis in the direction of the geomagnetic field appropriate to the particular current source being considered.

III. ELECTRON CURRENT AND DENSITY

The number of gamma rays or x rays emitted by a nuclear explosion per unit time is

$$\mathring{N}(t) = \frac{Y}{E} f(t) , \tag{1}$$

where Y is the yield of the explosion in the form of gamma rays or x rays, E is the mean gamma-ray or x-ray energy, and f(t) expresses the time variation of the gamma-ray or x-ray emission, normalized so that

$$\int_{-\infty}^{\infty} dt\, f(t) = 1 . \tag{2}$$

The rate at which primary electrons (Compton recoils and photoelectrons) are produced at a distance r in direction θ, φ from the explosion is

$$\mathring{n}_{pri}(\underline{r}, t) = g(\underline{r})\, f(t - r/c) , \tag{3}$$

where

$$g(\underline{r}) = \frac{Y}{E} \frac{e^{-\int dr/\lambda}}{4\pi r^2 \lambda} \tag{4}$$

and the integral extends from the burst point to the point under consideration. λ denotes the mean free path for production of primary electrons,

$$\lambda_\gamma = \lambda_{\gamma o}\, \rho_o/\rho \approx 3 \times 10^4\, \rho_o/\rho \text{ cm}, \tag{5}$$

$$\lambda_x = \lambda_{xo}\, \rho_o/\rho \approx \frac{1}{4} E_x^{-3}\, \rho_o/\rho \text{ cm}, \tag{6}$$

where ρ_o/ρ is the ratio of sea-level air density to the air density at the point of interest, and E_x is the x-ray energy in kilovolts. The primary electrons are slowed down by the atmosphere and are stopped in a distance given by their range, R , where

$$R_\gamma = R_{o\gamma}\, \rho_o/\rho \approx 3 \times 10^2\, \rho_o/\rho \text{ cm}, \tag{7}$$

$$R_x = R_{ox}\, \rho_o/\rho \approx 10^{-3} E_x^2\, \rho_o/\rho \text{ cm}. \tag{8}$$

For simplicity we shall assume that the electrons maintain their initial speed, v_o , throughout their range and then abruptly stop. At a particular point in space and time, the primary electron density, secondary electron densities, and currents are determined by adding the contributions from all primary electrons made earlier and which originated at appropriate positions and times. The detailed calculation is given in the Appendix. We give here only the results.

In the <u>absence of a magnetic field</u>, the primary electron density at position $\underline{r}$ and at time t is

$$n_{pri}(\underline{r}, t) \approx g(\underline{r}) \int_0^{R/v_o} d\tau' f\left[\tau - \tau'\left(1 - \frac{v_o}{c}\right)\right] , \tag{9}$$

where $\tau = t - r/c$. If we assume that the secondary electrons are made at a uniform rate during the slowing down of the primaries, then their rate of production is obtained from the density of primaries by multiplying by $\frac{v_o q}{R}$, where $q = \frac{E_{pri}}{33 ev}$ and E_{pri} is the initial energy of a primary electron. Thus

$$\mathring{n}_{sec}(\underline{r}, t) \approx \frac{v_o q}{R} \, g(\underline{r}) \int_0^{R/v_o} d\tau' f\left[\tau - \tau' (1 - v_o/c)\right], \qquad (10)$$

and the secondary electron density is then

$$n_{sec}(\underline{r}, t) \approx \frac{v_o q}{R} \, g(r) \int_{-\infty}^{\tau} d\tau' \int_0^{R/v_o} d\tau'' f\left[\tau' - \tau''(1 - v_o/c)\right]. \qquad (11)$$

With no magnetic field, the net electron current is in the radial direction and is

$$j_r(\underline{r}, t) \approx - e v_o g(\underline{r}) \epsilon \int_0^{R/v_o} d\tau' f\left[\tau - \tau'(1 - v_o/c)\right], \qquad (12)$$

where ϵ equals the mean cosine of the angle of scatter and is the effective fraction of primaries moving in the radial direction. For the photoelectric effect, $\epsilon \approx \frac{4}{5} \, v_o/c$ and for Compton scattering $\epsilon \approx 1$.

In the presence of a magnetic field, there are two modifica- tions to the above results. First, currents are generated in the transverse as well as the radial direction. Secondly, as the electrons are accelerated by the magnetic field their radial component of velocity is reduced which, in the gamma-ray case, causes the current to lag in time behind the gamma-ray pulse and which, in the x-ray case, causes an additional lag that is a small increment to the lag already caused by the low initial photoelectron velocities. The modified results, applicable both to the gamma rays and to the x rays, are

$$n_{sec}(\underline{r}, t) \approx q \frac{v_o}{R} g(\underline{r}) \int_{-\infty}^{\tau} d\tau' \int_0^{R/v_o} d\tau'' f\left[\tau' - (1-\beta\cos^2\theta)\tau'' + \beta\sin^2\theta \frac{\sin\omega\tau''}{\omega}\right] , \qquad (13)$$

$$j_r(\underline{r}, t) \approx -e\epsilon v_o g(\underline{r}) \int_0^{R/v_o} d\tau' f\left[\tau - (1-\beta\cos^2\theta)\tau' + \beta\sin^2\theta \frac{\sin\omega\tau'}{\omega}\right]\left[\cos^2\theta + \sin^2\theta\cos\omega\tau'\right] , \qquad (14)$$

$$j_\theta(\underline{r}, t) \approx -e\epsilon v_o g(\underline{r}) \int_0^{R/v_o} d\tau' f\left[\tau - (1-\beta\cos^2\theta)\tau' + \beta\sin^2\theta \frac{\sin\omega\tau'}{\omega}\right] \sin\theta\cos\theta\,(\cos\omega\tau' - 1) , \qquad (15)$$

$$j_\phi(\underline{r}, t) \approx -e\epsilon v_o g(\underline{r}) \int_0^{R/v_o} d\tau' f\left[\tau - (1-\beta\cos^2\theta)\tau' + \beta\sin^2\theta \frac{\sin\omega\tau'}{\omega}\right] \sin\theta\sin\omega\tau' , \qquad (16)$$

where for gamma rays, $\epsilon \approx 1$ and $\beta = v_o/c$, and for x rays, $\epsilon = \frac{4}{5}\beta$, and terms in the brackets involving $\beta = v_o/c << 1$ can be neglected. θ is the angle between the magnetic field and the direction of motion of the x rays and gamma rays, and ω is the Larmor frequency for the primary electrons, namely,

$$\omega = \frac{eB}{\gamma mc} = \frac{eB}{mc}\left(1 - v_o^2/c^2\right)^{1/2} . \qquad (17)$$

IV. ELECTROMAGNETIC FIELDS FROM HIGH-ALTITUDE CURRENTS

Equations (13) to (16) show that a nuclear explosion produces a current and electron density in the atmosphere which expands as a pulse at the speed of light radially from the source. The magnitude of the pulse is proportional to $g(\underline{r})$ of Eq. (4) which is large both near the source and in the earth's atmosphere, where most of the radiation is absorbed. As remarked before, we shall limit our considerations to the pulse produced in this latter region. The time behavior of the pulse is that of a function which varies rapidly with the retarded time $\tau = t - r/c$, where r is distance from the explosion. It is convenient, therefore, to rewrite Maxwell's equations in terms of the retarded time.

In the conventional form, Maxwell's equations are

$$\nabla \times \underline{B} = \frac{1}{c}\frac{\partial \underline{E}}{\partial t} + \frac{4\pi}{c}\underline{j} , \tag{18}$$

$$\nabla \times \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} , \tag{19}$$

$$\nabla \cdot \underline{B} = 0 , \tag{20}$$

$$\nabla \cdot \underline{E} = 4\pi\rho . \tag{21}$$

Using the transformations to retarded time $\tau = t - r/c$, namely,

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial \tau} , \tag{22}$$

$$\underline{\nabla}_t \rightarrow \underline{\nabla}_\tau - \underline{e}_r \frac{1}{c} \frac{\partial}{\partial \tau} , \tag{23}$$

where $\underline{e}_r$ is a unit vector directed radially from the explosion point, Maxwell's equations become

$$\nabla \times \underline{B} - \frac{1}{c} \underline{e}_r \times \frac{\partial \underline{B}}{\partial \tau} = \frac{1}{c} \frac{\partial \underline{E}}{\partial \tau} + \frac{4\pi}{c} \underline{j} , \tag{24}$$

$$\frac{1}{c} \frac{\partial \underline{B}}{\partial \tau} = -\nabla \times \underline{E} + \frac{1}{c} \underline{e}_r \times \frac{\partial \underline{E}}{\partial \tau}, , \tag{25}$$

$$\nabla \cdot \underline{B} - \frac{1}{c} \frac{\partial B_r}{\partial \tau} = 0 , \tag{26}$$

$$\nabla \cdot \underline{E} - \frac{1}{c} \frac{\partial E_r}{\partial \tau} = 4\pi\rho . \tag{27}$$

As usual, $\underline{j}$ represents the total current, including "back currents" or polarization currents in the medium. Conservation of charge, which follows from these equations, becomes

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot j - \frac{1}{c} \frac{\partial j_r}{\partial \tau} = 0 . \tag{28}$$

It is convenient to eliminate $\underline{E}$ from Maxwell's equations and get equations involving only $\underline{B}$, and conversely . Doing this, we obtain the following equations

$$-(\nabla^2\underline{B})_r - \frac{4\pi}{c}(\nabla \times \underline{j})_r + \frac{2}{c}\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right] = 0, \tag{29}$$

$$-(\nabla^2\underline{B})_\theta - \frac{4\pi}{c}(\nabla \times \underline{j})_\theta + \frac{2}{c}\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{2\pi}{c}j_\phi\right] = 0, \tag{30}$$

$$-(\nabla^2\underline{B})_\phi - \frac{4\pi}{c}(\nabla \times \underline{j})_\phi + \frac{2}{c}\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi) + \frac{2\pi}{c}j_\theta\right] = 0, \tag{31}$$

where $(\nabla^2\underline{B})_r$ is the radial component of $\nabla^2\underline{B}$, etc. Analogously,

$$-(\nabla^2\underline{E})_r + \frac{4\pi}{c}(\nabla . j) + 4\pi(\nabla\rho)_r + \frac{2}{c}\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_r)\right] = 0, \tag{32}$$

$$-(\nabla^2\underline{E})_\theta + 4\pi(\nabla\rho)_\theta + \frac{2}{c}\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) + \frac{2\pi}{c}j_\theta\right] = 0, \tag{33}$$

$$-(\nabla^2\underline{E})_\phi + 4\pi(\nabla\rho)_\phi + \frac{2}{c}\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) + \frac{2\pi}{c}j_\phi\right] = 0, \tag{34}$$

where

$$\frac{\partial\rho}{\partial\tau} = \frac{1}{c}\frac{\partial j_r}{\partial\tau} - \nabla . j . \tag{35}$$

We now observe that the variation of the currents with distance r is much slower than the variation of the currents with $c\tau$. This latter variation extends over times τ much less than a microsecond[4], corresponding to the emission times of the gamma rays and x rays from the explosion. The spatial variation of the currents extends over distances like the atmospheric scale height (~ 10 km),

[4] Latter, R., R. F. Herbst and K. M. Watson, Ann. Rev. of Nuc. Sci. II, 371 (1961).

which corresponds to times of tens of microseconds. For the transverse currents, the resultant electromagnetic fields have a rapidly varying time character similar to the current sources[5]. Thus, the time variation of the fields from the transverse currents is much more rapid than their spatial variation. We are thus led to an approximation in which we keep only the last bracket in Eqs. (30), (31), (33) and (34). That is,

$$\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)+\frac{2\pi}{c}j_\theta\right]\approx 0\,, \tag{36}$$

$$\frac{\partial}{\partial\tau}\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi)+\frac{2\pi}{c}j_\phi\right]\approx 0\,, \tag{37}$$

and similarly for the magnetic fields B_φ and B_θ. For the radial equations, to a similar order of approximation, we obtain a non-radiating field given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial E_r}{\partial\tau}+4\pi j_r\right)\right]\approx 0\,, \tag{38}$$

and

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial B_r}{\partial\tau}\right)-\frac{2\pi}{r\sin\theta}\frac{\partial\left(\sin\theta\, j_\varphi\right)}{\partial\theta}\approx 0\,, \tag{39}$$

[5]For the radial currents, the radiated field does not have a time character similar to the current sources. Rather its character is determined by the spatial extent of the current source. For the currents generated in the atmosphere, the spatial extent of the currents is very great and therefore the radial components contribute a very low frequency electromagnetic signal. This signal is, in general, weak relative to that due to the transverse currents and therefore may be neglected.

where we have approximately $\rho \approx \frac{1}{c}\, j_r$ from Eq. (28). Integrating these equations, we arrive at the final approximate equations [6]

$$\frac{\partial E_r}{\partial \tau} + 4\pi j_r = 0 , \tag{40}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_\theta) + \frac{2\pi}{c}\, j_\theta = 0 , \tag{41}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_\phi) + \frac{2\pi}{c}\, j_\phi = 0 , \tag{42}$$

with the corresponding results for the magnetic field. The first equation, Eq. (40), is the familiar one for the radial electric field. We shall not be concerned with this non-radiating component, but only with the propagating transverse fields.

We recall that the current density $\underline{j}$ is to include the secondary currents in the plasma as well as the primary electron currents. Thus,

$$\underline{j} = \underline{j}_{pri} + \underline{j}_{sec} . \tag{43}$$

To determine $\underline{j}_{sec}$ we use the Lorentz model. We assume that the plasma electrons are created at rest and that their motion is determined only by the local electric field and by their collisions with the air. Thus, if $\underline{x}(t)$ is the displacement of a plasma electron from its initial position, which we assume small compared to important wavelengths,

$$\ddot{\underline{x}} + \nu_c \dot{\underline{x}} = -\frac{e}{m}\, \underline{E} . \tag{44}$$

[6] Results analogous to Eqs. (41) and (42), but in a planar approximation where the factors of r are not included, have previously been obtained by C. Longmire (unpublished 1964).

For an electron created at time t_o, the current, $-e\dot{\underline{x}}$, is given by

$$-e\,\dot{\underline{x}}(t;t_o) = \frac{e^2}{m}\int_{t_o}^{t} dt'\,e^{-\nu_c(t-t')}\,\underline{E}(t')\,. \qquad (45)$$

If $\dot{n}_{sec}(t_o)$ is the rate at which the electrons are made, then the current density is given by

$$\underline{j}(t) = -\int_{-\infty}^{t} dt_o\,\dot{n}_{sec}(t_o)\,e\,\dot{\underline{x}}(t;t_o) \qquad (46)$$

or, from Eq. (45),

$$\underline{j}(t) = \frac{e^2}{m}\int_{-\infty}^{t} dt'\,e^{-\nu_c(t-t')}\,\underline{E}(t')\,n_{sec}(t')\,. \qquad (47)$$

Clearly, if the plasma electrons were disappearing, as well as being created, the same final expression would result— $n_{sec}(t)$ then being the net electron density. Thus, we are led to the following result for the plasma current

$$\underline{j}_{plasma}(\underline{r},t) = \frac{e^2}{m}\int_{-\infty}^{t} dt'\,e^{-\nu_c(\underline{r})\,[t-t']}\,\underline{E}(\underline{r},t')\,n_{sec}(\underline{r},t')\,. \qquad (48)$$

If we substitute this expression for the plasma current into the field equations, we obtain for E_θ and E_ϕ

$$\frac{\partial}{\partial r}\left[r\,E_\theta(\underline{r},\tau)\right] + \frac{2\pi}{c}\,\frac{e^2}{m}\int_{-\infty}^{\tau} d\tau'\, r\,E_\theta(\underline{r},\tau')\,n_{sec}(\underline{r},\tau')\,e^{-\nu_c(\underline{r})(\tau-\tau')}$$

$$+\frac{2\pi}{c}\,r\,j_\theta(\underline{r},\tau) = 0\,, \tag{49}$$

$$\frac{\partial}{\partial r}\left[r\,E_\varphi(\underline{r},\tau)\right] + \frac{2\pi}{c}\,\frac{e^2}{m}\int_{-\infty}^{\tau} d\tau'\, r\,E_\varphi(\underline{r},\tau')\,n_{sec}(\underline{r},\tau')\,e^{-\nu_c(\underline{r})(\tau-\tau')}$$

$$+\frac{2\pi}{c}\,r\,j_\varphi(\underline{r},\tau) = 0 \tag{50}$$

and similar results for B_θ and B_φ. These equations are not, in general, soluble because of the spatial dependence of $\nu_c(\underline{r})$ in the exponential.

However, if $\nu_c(\underline{r})$ is large compared to the frequencies in E and n_{sec}, the equations simplify to

$$\frac{\partial}{\partial r}(r\,E) + \frac{2\pi}{c}\,\sigma\, r\,E + \frac{2\pi}{c}\,r\,j = 0\,, \tag{51}$$

where it is understood that $E \to E_{\theta,\varphi}$ and $j \to j_{\theta,\varphi}$. This equation is readily soluble and yields

$$E(\underline{r},\tau) = -\frac{2\pi}{c}\,\frac{1}{r}\int_0^r dr'\,r'\,j(\underline{r}',\tau)\,e^{-\frac{2\pi}{c}\int_{r'}^{r} dr''\,\sigma(\underline{r}'',\tau)}\,, \tag{52}$$

where $\sigma(\underline{r},t)$ is given by

$$\sigma(\underline{r},t) = \frac{e^2}{m\nu_c}\,n_{sec}(\underline{r},t)\,. \tag{53}$$

In the opposite extreme, for a collisionless plasma with $\nu_c(\underline{r}) = 0$, the equations simplify to

$$\frac{\partial}{\partial r}(r E) + \frac{2\pi e^2}{mc} r \int_{-\infty}^{\tau} d\tau' \, n_{sec}(\underline{r}, \tau') \, E(\underline{r}, \tau') + \frac{2\pi}{c} r \, j(\underline{r}, \tau) = 0, \quad (54)$$

which has the solution[7]

$$E(\underline{r}, \tau) = -\frac{2\pi}{c} \frac{1}{r} \int_{0}^{r} dr' \int_{-\infty}^{\tau} d\tau' \, r' \frac{\partial j}{\partial \tau} (\underline{r}', \tau')$$

$$J_0 \left[\left(\frac{8\pi e^2}{mc} \int_{\tau'}^{\tau} d\tau'' \int_{r'}^{r} dr'' \, n_{sec}(\underline{r}'', \tau'') \right)^{1/2} \right]. \quad (55)$$

Analogous results for the magnetic field are readily derived. In particular, it is easily shown that $B_\varphi = E_\theta$ and $B_\theta = -E_\varphi$

Thus, to obtain the transverse electric field from a pulse of transverse current moving with light speed, we add all the contributions along the line from the explosion to the observer. In the conducting plasma these contributions are reduced by absorption between the source and the observer; in the collisionless plasma, dispersion is introduced by the plasma.

---

[7] If ν_c is independent of position, it is easy to show that the solution of Eqs. (49) and (50) is obtained from Eq. (55) by replacing $E(\underline{r}, t)$ with $E(\underline{r}, t) e^{\nu_c t}$ and $j(\underline{r}, t)$ by $j(\underline{r}, t) e^{\nu_c t}$

V. DETECTION CONSIDERATIONS

It is clear from Eqs. (52) and (55) that the electromagnetic field observed from high–altitude nuclear explosions depends only upon the distribution of current and electron density along the line between the burst point and the observation point. However, because of the exponential variation of atmospheric density, the magnitude of the electromagnetic field depends quite sensitivity upon just where the currents and electron densities are generated in the atmosphere. Moreover, the occurrence of the conductivity $\sigma(\underline{r}, \tau)$ in the exponential of Eq. (52) indicates that the magnitude of the electron density at each position in the atmosphere is also quite important. The result is that the magnitude of the observed electromagnetic field is quite sensitive to the gamma–ray and x–ray yield of the explosion, the distance between the observation point and the burst point, the temperature of the explosion, and the angle of incidence of the radiation on the atmosphere[8]. A comprehensive description of the electromagnetic field would therefore require a parametric study for beyond the intent of this paper which is simply to indicate the possible usefulness of this signal for detection. What we shall do

---

[8]Effects due to the earth's ionosphere are not important in the present considerations, since the currents which generate the electromagnetic signal are produced below the ionosphere. For a detailed discussion of the deposition of explosion x rays and gamma rays in the atmosphere, see R. Latter and R. E. LeLevier, Journ. Geophys. Res. 68, 1643 (1963).

here is simply indicate the crude order of magnitude of the electromagnetic field strength under typical explosion conditions and then indicate roughly the limiting detection distance.

First, we shall consider explosions which are not too distant from the earth, which have a temperature of about one kev, and for which radiation is incident almost normally on the atmosphere. Specifically, these conditions mean that the ionization produced deep in the atmosphere by the x rays and gamma rays is high, so that ν_c in the regions of interest[9] is large, allowing the approximation Eq. (52) to be applied, and, moreover,

$$\frac{2\pi}{c} \int_0^r dr' \, \sigma(r', \tau) \gtrsim 1 . \qquad (56)$$

This condition implies[8] for x rays that a one kiloton explosion is within about a few times 10^4 km from the earth's surface and for gamma rays, within about a few times 10^2 km.

---

[9]For low pressures, large electric fields and electron–volt energies, the collision frequency of electrons in air is approximately independent of the E–field and has a value $\nu_c = 4 \times 10^{12} \rho/\rho_o \text{ sec}^{-1}$; see R. A. Nielsen and N. E. Bradbury, Phys. Rev. 51, 69 (1937).

To estimate the magnitude of this radiation, we first simplify Eq. (52). We note that the factor r' in the integrand of Eq. (52) can be replaced by r, the distance to the observation point, since the important currents are confined to a thin layer in the atmosphere near the observation point. Thus

$$E_{\theta,\varphi}(r,\tau) \approx -\frac{2\pi}{c}\int_0^r dr'\, j_{\theta,\varphi}(r',\tau)\, e^{-\frac{2\pi}{c}\int_{r'}^{r} dr''\sigma(r'',\tau)} \quad . \tag{57}$$

Now we note that for r' sufficiently near r,

$$\frac{2\pi}{c}\int_{r'}^{r} dr''\sigma(r'',\tau) \approx 0 \quad \text{and} \quad j(r',\tau) \approx 0 \ , \tag{58}$$

since the gamma rays and x rays are rapidly absorbed by the atmosphere below some low altitude. As r' decreases and the altitude increases, we finally reach a radius r_o at which the x-ray and gamma-ray currents and ionization increase rapidly. Thus we can rewrite Eq. (58) as

$$E_{\theta,\varphi}(r,\tau) \approx -\frac{2\pi}{c}\int_0^{r_o} dr'\, j_{\theta,\varphi}(r',\tau)\, e^{-\frac{2\pi}{c}\int_{r'}^{r_o} dr''\sigma(r'',\tau)} \ , \tag{59}$$

or making an obvious change of variables,

$$E_{\theta,\varphi}(r,\tau) \approx -\int_0^{\frac{2\pi}{c}\int_0^{r_o} dr''\sigma(r'',\tau)} dx\, \frac{j_{\theta,\varphi}(r'(x),\tau)}{\sigma(r'(x),\tau)}\, e^{-x} \ . \tag{60}$$

It is easy to show that $\frac{j_{\theta,\varphi}(r'(x),\tau)}{\sigma(r'(x),\tau)}$ varies slowly compared to e^{-x}

and therefore[10]

$$E_{\theta,\varphi}(r,\tau) \approx - \frac{j_{\theta,\varphi}(r_o,\tau)}{\sigma(r_o,\tau)} \left(1 - e^{-\frac{2\pi}{c}\int_0^{r_o} dr''\sigma(r'',\tau)}\right),$$

$$\approx - \frac{j_{\theta,\varphi}(r_o,\tau)}{\sigma(r_o,\tau)} . \qquad (61)$$

Substituting Eqs. (15), (16) and (53) into Eq. (61), we find

$$E_{\theta,\varphi} = \frac{\epsilon m\omega \nu_c R}{eq} \frac{J_{\theta,\varphi}(\tau)}{N(\tau)} , \qquad (62)$$

where

$$N(\tau) = \omega \int_{-\infty}^{\tau} d\tau'' \int_0^{R/v_o} d\tau' f\left[\tau - (1-\beta\cos^2\theta)\tau' + \beta\sin^2\theta \frac{\sin\omega\tau'}{\omega}\right] , \qquad (63)$$

$$J_\theta(\tau) = \int_0^{R/v_o} d\tau' f\left[\tau - (1-\beta\cos^2\theta)\tau' + \beta\sin^2\theta \frac{\sin\omega\tau'}{\omega}\right] \sin\theta\cos\theta\,(\cos\omega\tau' - 1) , \qquad (64)$$

$$J_\varphi(\tau) = \int_0^{R/v_o} d\tau' f\left[\tau - (1-\beta\cos^2\theta)\tau' + \beta\sin^2\theta \frac{\sin\omega\tau'}{\omega}\right] \sin\theta\sin\omega\tau' . \qquad (65)$$

We note that for gamma rays, which deposit most of their energy in the altitude range from 20 to 40 km,

$$\frac{R}{v_o} \sim 2 \times 10^{-7} \text{ to } 3 \times 10^{-6} \text{ sec.} \qquad (66)$$

---

[10] Equation (61) gives the result obtained by a more direct calculation in the paper cited in footnote 2.

Since the gamma rays are emitted in times[4] of about 10^{-7} sec, we see that order–of–magnitude–wise $f(\tau)$ can be replaced by a delta function. Similarly, for x rays which are emitted in times[4] considerably less than 10^{-6} sec, and are absorbed in the altitude range from 70 to 110 km,

$$\frac{R}{v_o} \sim 10^{-7} \text{ to } 5 \times 10^{-5} \text{ sec}, \tag{67}$$

where $R \sim 10^{-2} \rho/\rho_o$ cm for electrons with energies of a few kilovolts, so that again $f(\tau)$ can be replaced approximately by a delta function.

Thus,

$$\begin{aligned} N(\tau) &\approx X , & X &< \frac{\omega R}{\beta c} , \\ &\approx \frac{\omega R}{\beta c} , & X &> \frac{\omega R}{\beta c} , \end{aligned} \tag{68}$$

$$\begin{aligned} J_\theta(\tau) &\approx - \frac{\sin\theta\cos\theta\,(1-\cos X)}{1-\beta\cos^2\theta-\beta\sin^2\theta\cos X} , & X &< \frac{\omega R}{\beta c} , \\ &\approx 0 , & X &> \frac{\omega R}{\beta c} , \end{aligned} \tag{69}$$

$$\begin{aligned} J_\varphi(\tau) &\approx \frac{\sin\theta\sin X}{1-\beta\cos^2\theta-\beta\sin^2\theta\cos X} , & X &< \frac{\omega R}{\beta c} , \\ &\approx 0 , & X &> \frac{\omega R}{\beta c} , \end{aligned} \tag{70}$$

where X is determined from

$$(1-\beta\cos^2\theta)\,X - \beta\sin^2\theta\sin X \approx \omega\tau . \tag{71}$$

Numerical evaluation of $J_{\theta,\varphi}/N$, using $\beta = v_o/c = 0.95$ corresponding to 1 mev Compton–recoil electrons and $\beta = v_o/c = 0.05$ to 0.1 corresponding to 1 to a few kev x–ray photoelectrons, gives the peak

$J_{\theta,\varphi}/N \sim 1/(1-\beta)$. Hence, order–of–magnitude–wise

$$E_{\theta,\varphi} \sim \frac{\epsilon m \omega \nu_c R}{eq(1-\beta)} \ . \tag{72}$$

For gamma rays wherein $\epsilon \approx 1$, $m \approx 9 \times 10^{-28}$ gm, $\omega \approx 1.67 \times 10^{6}$ cps, $e \approx 4.8 \times 10^{-10}$, $q \approx 3 \times 10^{4}$, and $R \approx 3 \times 10^{2}$ ρ/ρ_o cm, we have

$$E_{\theta,\varphi} \sim 2 \text{ esu or } 6 \times 10^{4} \text{ v/m} \ . \tag{73}$$

And for x rays, wherein[11] $\epsilon \approx 0.1$, $\omega \approx 5.3 \times 10^{6}$ cps, $q \approx 10^{2}$, and $R \approx 10^{-2}$ ρ/ρ_o,

$$E_{\theta,\varphi} \sim 3.5 \times 10^{-4} \text{ esu or } 10 \text{ v/m} \ . \tag{74}$$

Both of these fields are sufficiently intense so as to be easily detectable.[12]

We now turn to the problem of estimating the intensity of the electromagnetic signal from explosions at extreme distances from

---

[11] A mean photoelectron energy of about 3 kev is assumed, since the x rays which contribute most to $E_{\theta,\varphi}$ are those which penetrate deepest. For the range of low–energy electrons, see Eq. (8) and L. Katz and A. S. Penfold, Rev. Mod. Phys. 24, 28 (1952).

[12] Actually, for the assumed close distance of the explosions and the assumed normal incidence of the radiations on the atmosphere, the pre–ionization produced by the gamma rays is sufficiently intense to absorb the x–ray produced signal before it reaches the earth's surface. Thus, under these circumstances, only the gamma–ray produced signal is useful for detection.

the earth, and thereby obtain the limit of detection. For sufficiently great distances, we may neglect n_{sec} and σ in Eqs. (52) and (55), so that

$$E_{\theta,\varphi}(r,\tau) \approx -\frac{2\pi}{c}\frac{1}{r}\int dr'r'j_{\theta,\varphi}(r',\tau)\,, \tag{75}$$

and the integration is over only the regions in the atmosphere near the earth's surface where gamma rays and x rays are deposited.

Substituting Eqs. (4), (15) and (16) into Eq. (75) and making the delta-function approximation for $f(\tau)$, we find

$$E_{\theta,\varphi}(r,\tau) = \frac{e\beta\epsilon Y}{2E} J_{\theta,\varphi}(\tau)\frac{1}{r}\int^{r} dr' \frac{e^{-\int^{r'} dr''/\lambda}}{r'\lambda}\,, \tag{76}$$

where $J_{\theta,\varphi}(\tau)$ is defined by Eqs. (69) and (70). Since the integrations extend over a region small compared to r' and since the integral $\int \frac{dr'}{\lambda}$ over the important region is large,

$$E_{\theta,\varphi}(r,\tau) \approx \frac{e\beta\epsilon}{2E} J_{\theta,\varphi}(\tau)\frac{Y}{r^2}\,. \tag{77}$$

Finally, it is clear that $J_{\theta,\varphi}(\tau) \sim 1$, so that

$$E_{\theta,\varphi} \sim \frac{e\beta\epsilon}{2E}\frac{Y}{r^2}\,. \tag{78}$$

For gamma rays,

$$E_{\theta,\varphi} \sim 6 \times 10^{2} \frac{Y_{KT}}{r^{2}_{km}} \text{ esu} , \tag{79}$$

where the fraction[8] of the total yield Y_{KT} emitted as gamma rays is taken to be 10^{-3}. For x rays,

$$E_{\theta,\varphi} \sim 2 \times 10^{5} \frac{Y_{KT}}{r^{2}_{km}} \text{ esu} , \tag{80}$$

where half the total yield[4] Y_{KT} is assumed to be emitted as x rays.

To estimate the maximum detection range of this signal, we shall assume that the only interfering noise is due to the cosmic noise background. Thus, it will be assumed that the detectors can be effectively shielded from man–made disturbances.

The mean cosmic noise power is

$$P_N = 4\pi kT/\lambda^2 = 1.7 \times 10^{-22} \, \sigma T/\lambda^2 \text{ w/m}^2 , \tag{81}$$

where T is the effective temperature in degrees Kelvin of the cosmic noise at the wavelength λ in meters and σ is the detector bandwidth in cps. Since the signal to be detected has frequencies from one to ten megacycles (See Eqs. (66), (67), (69), (70) and (71)), the detector bandwidth should be adjusted to $\sigma \sim 10^7$ cps at $\lambda \sim 10^2$ m. Under these circumstances the effective temperature of the cosmic noise background is $T \sim 10^6$ °K. Hence, the interfering noise power is

$$P_N \approx 1.7 \times 10^{-13} \ w/m^2 . \tag{82}$$

For reliable detection—using coincidence techniques—we require the peak signal power P_s to exceed the noise power by a factor of ten, that is,

$$P_s \approx \frac{c}{4\pi} E_s^2 = 10 \ P_N . \tag{83}$$

Or

$$E_s \approx 8 \times 10^{-10} \ \text{esu} \quad \text{or} \quad 2.4 \times 10^{-5} \ v/m . \tag{84}$$

Thus, from Eqs. (79) and (80), the limiting detection range for gamma rays is

$$r_{km} \sim 0.8 \times 10^6 \sqrt{Y_{KT}} , \tag{85}$$

and for x rays is

$$r_{km} \sim 1.5 \times 10^7 \sqrt{Y_{KT}} . \tag{86}$$

VI. DISCUSSION

An interesting consequence of Eqs. (72) to (74) and Eq. (78) to (80) is that for near explosions, the gamma–ray generated electro–magnetic signal is more intense than the x–ray generated signal, while for distant explosions the opposite is true. To understand this result we note that for distant explosions the intensity of the radiated field is proportional to the yield of the explosion appearing as gamma rays or x rays—thereby, as would be expected, the x–ray generated signal should be larger. As the burst point comes nearer to the earth's atmosphere, the intensity of the signal begins to level off and becomes independent of yield. For x rays, this leveling off occurs at a very much greater distance than for gamma rays—thus the smaller intensity of the x–ray generated signal.

A final point—in concluding that the detection range of the x–ray generated signal was given by Eq. (86) and exceeded the gamma–ray generated signal, it was assumed that the x–ray yield of the explosion was about one–half the total yield. Actually, however, this yield might be considerably less—perhaps by a factor of 10^2 to 10^3, if shielding were employed[4]. In this case the limiting detection range would be determined by the gamma–ray generated signal. Shielding of the gamma–ray signal might also be possible, thereby degrading the detection range even further.

APPENDIX

In this appendix we derive the formulae for the primary electron current and secondary electron density as functions of space and time.

The rate of production of primary electrons at a point $\underline{r}'$, at time t', with initial motion in direction $\underline{\Omega}$, is given by

$$\mathring{n}_{pri}(t', \underline{r}'; \underline{\Omega}) = g(\underline{r}')\, f(t - \frac{\underline{r}'}{c})\, h(\underline{\Omega}) , \qquad \text{(A–1)}$$

where the angular distribution $h(\underline{\Omega})$ is so normalized that

$$\int d\underline{\Omega}\ h(\underline{\Omega}) = 1 , \qquad \text{(A–2)}$$

$$\int d\underline{\Omega}\ \Omega_r\, h(\underline{\Omega}) = \epsilon , \qquad \text{(A–3)}$$

and Ω_r is the cosine of the angle between the direction of motion of the electrons and the initial gamma–ray or x–ray direction. For gamma–ray produced Compton electrons, $\epsilon \approx 1$. For photoelectrons from x rays, $\epsilon = \frac{4}{5}\, v_o/c \approx 0.05$ to 0.1 for 1 to a few kev electrons. From symmetry, the other components of $\underline{\Omega}$ integrate to zero.

The primary electron current and density at a position $\underline{r}$ and time t are obtained by adding the contributions from those electrons made at earlier times and appropriate positions such that their motion brings them to $\underline{r}$ at time t . That is,

$$j(\underline{r}, t) = -e \int d\underline{r}' \int dt' \int d\underline{\Omega}\, g(\underline{r}')\, f\left(t' - \frac{r'}{c}\right) h(\underline{\Omega})\, \underline{v}(t, t'; \underline{\Omega})$$

$$\delta\left[\underline{r} - \underline{r}' - \int_{t'}^{t} dt''\, \underline{v}(t'', t'; \underline{\Omega})\right], \qquad \text{(A–4)}$$

where $\underline{v}\,(t, t'; \underline{\Omega})$ is the velocity at time t of an electron made at t' with initial direction $\underline{\Omega}$. The density of primary electrons is given by the same expression without the factor $-e\underline{v}$. In the absence of a magnetic field or any slowing down mechanism, the velocity of the electrons is constant, so that

$$\underline{v}\,(t, t'; \underline{\Omega}) = v_o \underline{\Omega} \quad , \qquad \text{(A–5)}$$

$$\int_{t'}^{t} dt''\, \underline{v}\,(t'', t'; \underline{\Omega}) = v_o \underline{\Omega}\,(t - t') \; . \qquad \text{(A–6)}$$

Since the electrons have a finite lifetime given by their range R divided by their initial speed, the integral over t' in Eq. (A–4) extends only over the finite range from $t - R/v_o$ to t . We shall assume that the distances the electrons travel are small compared to r, which is the case for currents generated in the atmosphere by high–altitude nuclear explosions. Thus, the direction from the source to $\underline{r}'$ can be taken to be the same as that to $\underline{r}$ and the current simplifies to

$$\underline{j}\,(\underline{r}, t) \approx -e v_o g(\underline{r}) \int dr' \int_{t-R/v_o}^{t} dt' f\left(t' - \frac{r'}{c}\right) \int d\underline{\Omega}\, \underline{\Omega}\, h(\underline{\Omega})$$

$$\delta\left[r - r' - v_o(t - t')\, \Omega_r\right] . \qquad \text{(A–7)}$$

With $t - t' \equiv \tau'$, $t - r/c \equiv \tau$, we have

$$\underline{j}(\underline{r}, t) = -ev_o g(\underline{r}) \int_o^{R/v_o} d\tau' \int d\underline{\Omega}\, h(\underline{\Omega})\, \underline{\Omega}\, f\left(\tau - \tau' + \frac{v_o \tau'}{c} \Omega_r\right). \qquad (A-8)$$

For Compton electrons $h(\underline{\Omega})$ is essentially a delta function in the initial direction. Thus the current is radial and

$$j_r(\underline{r}, t) \approx -ev_o g(\underline{r}) \int_o^{R/v_o} d\tau' f\left[\tau - \tau'(1 - \frac{v_o}{c})\right], \qquad (A-9)$$

$$n_{pri}(\underline{r}, t) \approx g(\underline{r}) \int_o^{R/v_o} d\tau' f\left[\tau - \tau'(1 - \frac{v_o}{c})\right]. \qquad (A-10)$$

For photoelectrons, we expand the function f in powers of

$\frac{v_o \tau'}{c} \Omega_r < \frac{R}{c} \Omega_r$. Thus

$$f(\tau - \tau' + \frac{v_o \tau'}{c} \Omega_r) \approx f(\tau - \tau') + \frac{v_o \tau'}{c} \Omega_r \mathring{f}(\tau - \tau') + \ldots \qquad (A-11)$$

The integration over $\underline{\Omega}$ in (A-8) converts Eq. (A-11) into an expansion in terms of $\frac{v_o \tau' \epsilon}{c}$ which is bounded by $\frac{R\epsilon}{c}$ and which is small compared to one at all altitudes of importance. Thus for photoelectrons,

$$j_r(\underline{r}, t) \approx -e\epsilon v_o g(\underline{r}) \int_o^{R/v_o} d\tau' f(\tau - \tau'), \qquad (A-12)$$

$$n_{pri}(\underline{r}, t) \approx g(\underline{r}) \int_o^{R/v_o} d\tau' f(\tau - \tau'). \qquad (A-13)$$

We note that although different approximations have been used, we can write the results in one form for both Compton electrons and photoelectrons

$$j_r(\underline{r}, t) \approx -e\epsilon v_o g(\underline{r}) \int_0^{R/v_o} d\tau' f\left[\tau - \tau'(1 - \frac{v_o}{c})\right], \qquad \text{(A-14)}$$

$$n_{pri}(\underline{r}, t) = -j_r/e\epsilon v_o, \qquad \text{(A-15)}$$

where $\epsilon \approx 1$ for Compton electrons, and $\epsilon = \frac{4}{5}\frac{v_o}{c}$ and the term $\frac{v_o}{c} << 1$ in the argument of f should be neglected for photoelectrons.

In the presence of a magnetic field, the electrons spiral about the field so that there are transverse as well as radial currents. Moreover, the radial velocity of the electrons is not constant with time but varies. Nonetheless, the earlier approximations are clearly still valid. Thus,

$$\underline{j}(\underline{r}, t) \approx -e\epsilon g(\underline{r}) \int_0^{R/v_o} d\tau' \underline{v}(\tau') f\left[\tau - \tau' + \frac{x(\tau')}{c}\right], \qquad \text{(A-16)}$$

$$n_{pri}(\underline{r}, t) \approx g(\underline{r}) \int_0^{R/v_o} d\tau' f\left[\tau - \tau' + \frac{x(\tau')}{c}\right], \qquad \text{(A-17)}$$

where $\underline{v}(\tau')$ is the velocity of an electron at time τ' after it is made and $x(\tau')$ is its displacement in the radial direction (direction of the initial gamma ray or x ray) from its initial position. As before,

$\epsilon \approx 1$ for Compton electrons, and we drop $x(\tau)/c$ in the argument of f for photoelectrons.

If only a static magnetic field is present, the motion of the recoil electrons is very simple. The component of velocity along the magnetic field is unchanged and the motion in the plane perpendicular to the field is circular with angular frequency

$$\omega = \frac{eB}{\gamma mc}, \tag{A-18}$$

where

$$\gamma = \left[1 - \left(\frac{v_o}{c}\right)^2\right]^{-1/2} \equiv (1-\beta^2)^{-1/2} \tag{A-19}$$

In terms of spherical coordinates,

$$x(\tau) = v_o \left[\cos^2\theta\, \tau + \sin^2\theta\, \frac{\sin \omega\tau}{\omega}\right], \tag{A-20}$$

$$v_r(\tau) = v_o \left[\cos^2\theta + \sin^2\theta \cos \omega\tau\right], \tag{A-21}$$

$$v_\theta(\tau) = -v_o \sin\theta \cos\theta\, (1 - \cos\omega\tau), \tag{A-22}$$

$$v_\phi(\tau) = v_o \sin\theta \sin\omega\tau, \tag{A-23}$$

where θ is the angle between the radial direction and the magnetic field. When these expressions are substituted into Eqs. (A-16) and (A-17), we obtain the results given in Section III, Eq. (13) to (16).

We have neglected several effects in deriving these expressions. First, we have assumed that the electrons have a constant speed, and then abruptly stop after an appropriate time, R/v_o. Actually, of course, they slow down as they lose energy by ionizing the air. Since the rate of energy loss, $F(\gamma)$, is known for electrons, it can, of course, be included. We then have to solve simultaneously the equations of motion

$$\frac{dE}{d\tau} = mc^2 \frac{d\gamma}{d\tau} = -F(\gamma) , \tag{A-24}$$

$$\frac{d}{d\tau} (\gamma m \underline{v}) = - \frac{e \underline{v} x \underline{B}}{c} - \frac{F(\gamma)}{v^2} \underline{v} . \tag{A-25}$$

To do this, we note that a straightforward integration of Eq. (A-24) gives

$$\int_{\gamma_o}^{\gamma} \frac{d\gamma}{F(\gamma)} = - \frac{\tau}{mc^2} , \tag{A-26}$$

where $\gamma_o = \gamma(\tau = 0)$. Solving this equation for γ as a function of τ we obtain the electron speed

$$v(\tau) = c \left[1 - \frac{1}{\gamma^2(\tau)} \right]^{1/2} . \tag{A-27}$$

Further, letting $\underline{v} \equiv v \underline{\hat{u}}$, where $\underline{\hat{u}}$ is the unit vector in the direction of the velocity, we find

$$\frac{d\underline{\hat{u}}}{d\tau} = - \frac{e}{vmc} \underline{\hat{u}} \times \underline{B} = -\omega(\tau) \hat{u} \times \frac{\underline{B}}{B} , \tag{A-28}$$

or

$$\frac{d\underline{\hat{u}}}{d\Theta} = - \frac{\underline{\hat{u}} \times \underline{B}}{B} , \tag{A-29}$$

where

$$\Theta(\tau) = \int_0^{\tau} d\tau' \, \omega(\tau') = \frac{eB}{mc} \int_0^{\tau} \frac{d\tau'}{\gamma(\tau')} = eBc \int_{\gamma(\tau)}^{\gamma_o} \frac{d\gamma}{\gamma F(\gamma)} . \tag{A-30}$$

Thus the electron velocities are the same as found before, with v_o replaced by $v(\tau) = c \left[1 - \frac{1}{\gamma^2(\tau)} \right]^{1/2}$, and $\omega\tau$ replaced by $\Theta(\tau)$.

The net effect is to reduce the velocity to zero during the slowing down and to increase the Larmor frequency ω as γ decreases to one.

Of greater concern is the question—what magnetic field should be used in determining the electron motion? Under conditions where the currents generate only weak fields, we need use the ambient magnetic field only. However, under conditions where the explosions are not too distant, the currents can give rise to intense crossed E and B fields which can significantly affect the motion of the electrons. For x-ray photoelectrons, however, the fields even under these conditions are sufficiently small so that only the ambient magnetic field need be used. For Compton-recoil electrons, the fields are large so that these self-generated fields must be considered. To estimate the effect of these Compton-current generated fields, we note first that the electric field produced by the electrons is transverse and in such a direction as to oppose the transverse motion of the electrons. The generated magnetic field, however, is in such a direction as to add to the ambient geomagnetic field and increase the transverse acceleration. The intensities of the generated electric and magnetic fields, which are equal in magnitude, depend upon the amount of transverse current and are time dependent. It is clearly impractical to try to solve rigorously the self-consistent problem for the electron motions. What we can do first is to observe that the total force on an electron is

$$\underline{F} = -e\left[\underline{E} + \frac{\underline{v}}{c} \times (\underline{B} + \underline{B}_o)\right] , \qquad \text{(A-31)}$$

where E and B are the fields due to the currents and $\underline{B}_o$ is the ambient field. Since $|\underline{E}| = |\underline{B}|$, the net transverse force is proportional to $E\,(1-\frac{v_r}{c})$ plus the transverse component of $\frac{\underline{v}}{c} \times \underline{B}_o$. If the radial electron velocity, v_r, is approximately light speed, the effect of the induced fields is greatly reduced. For Compton electrons, then, the initial turning is due primarily to the ambient field.

We can next approximately solve the self-consistent problem for Compton electrons by taking a simple special case. We let the gamma-ray source vary as a delta function in time, and take the initial direction of the electron motion at right angles to the ambient field. Then the equations of motion are

$$\frac{d}{d\tau}(\gamma \mathring{x}) = -\frac{eB_o}{mc}\mathring{y} - \frac{e}{mc}E(\tau)\,\mathring{y}\,, \tag{A-32}$$

$$\frac{d}{d\tau}(\gamma \mathring{y}) = \frac{eB_o}{mc}\mathring{x} - \frac{e}{m}E(\tau)\,(1-\frac{\mathring{x}}{c})\,, \tag{A-33}$$

where x is the direction of initial motion and y is the transverse direction. For not-too-distant explosions, for which the fields are highest, we find from Eqs. (61), (A-16) and (A-17) that

$$E(\tau) \approx -\frac{j}{\sigma} = \frac{m\nu_c R}{eq\beta c}\;\frac{\mathring{y}(\tau)}{\tau\left[1-\frac{\mathring{x}(\tau)}{c}\right]}\;. \tag{A-34}$$

It is straightforward to show that the maximum field strength occurs at $\tau = 0$ and has the value

$$E(0) = B_o\,\frac{\nu_c R}{q\,c(1-\beta)}\;\frac{1}{\gamma + \frac{\nu_c R}{\beta q\,c}}\;. \tag{A-35}$$

If we neglect the generated field in Eqs. (A-32) and (A-33), and consider the motion only in the ambient geomagnetic field, we find the peak value of the electric field again occurs at $\tau = 0$ and is given by

$$E(0) = B_o \frac{\nu_c R}{\gamma q c(1-\beta)} \quad . \qquad \text{(A-36)}$$

Thus the effect of the generated fields is to decrease the peak field by the factor

$$\gamma \left[\gamma + \frac{\nu_c R}{\beta q c} \right]^{-1} \approx 0.7 \; , \qquad \text{(A-35)}$$

where parameter values corresponding to 1 mev electrons have been used.

This example indicates that as long as the peak radiated field strength occurs near the beginning of the current pulse, the effect of the generated fields on the electron motion does not significantly change the order of magnitude of the estimate for the peak radiated field. The generated fields do affect the time variation of the radiated field. But since the range of detection depends principally on the peak field strength, the effect of the generated fields on the electron motion may be disregarded.

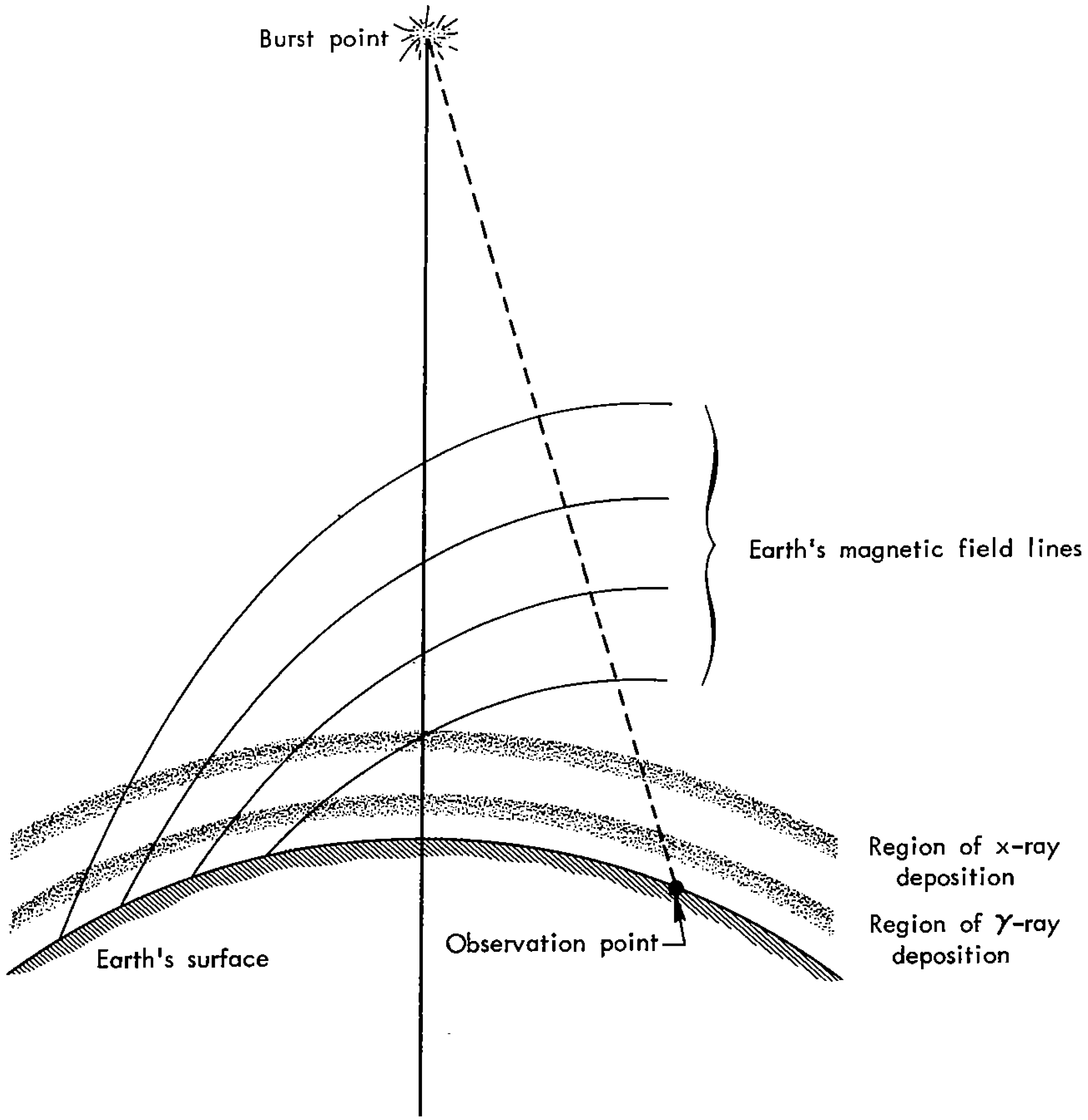

Figure 1

GEOMETRY OF THE EXPLOSION

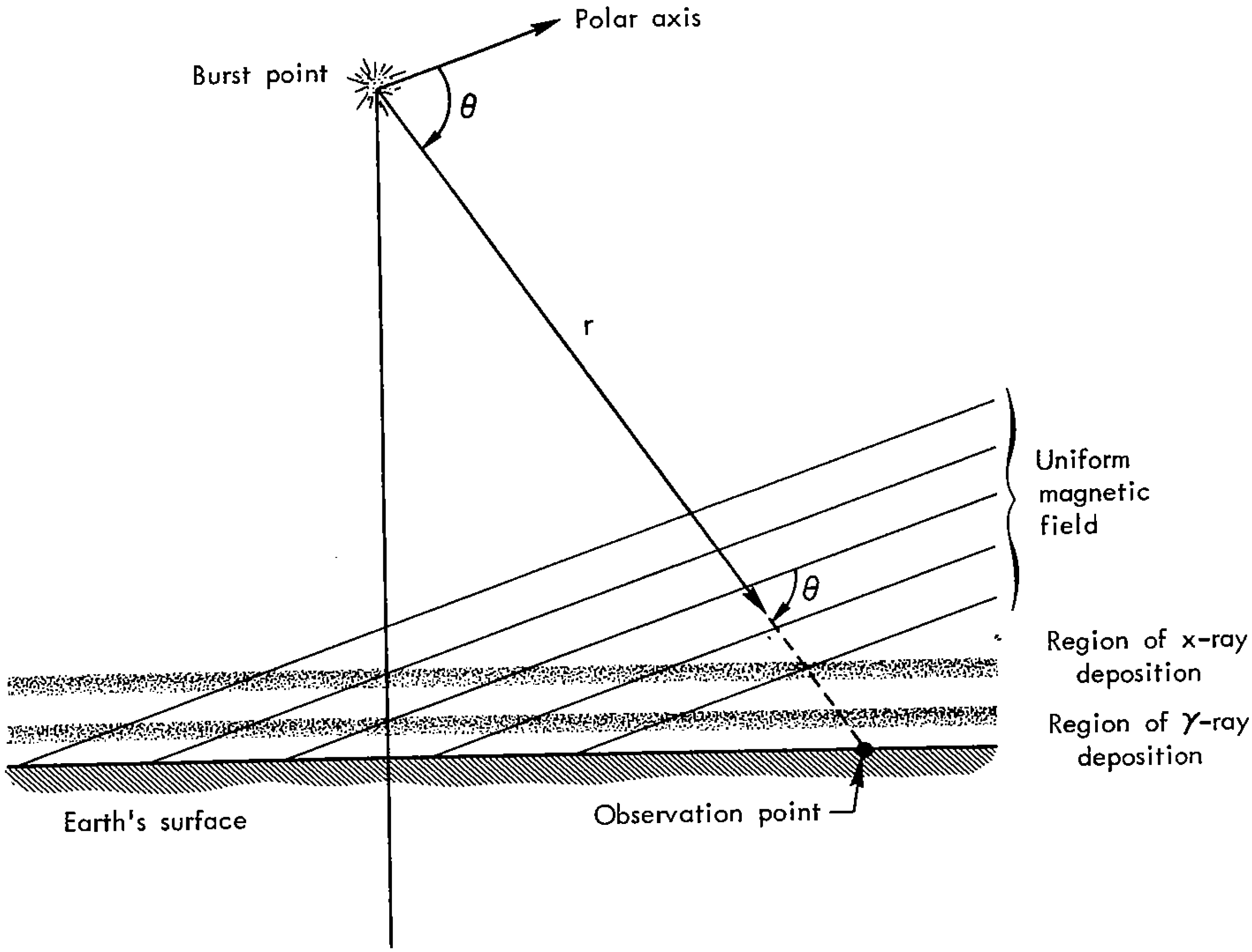

Figure 2

SIMPLIFIED GEOMETRY OF THE EXPLOSION